

ABOUT SPACE-TIME ASYMMETRY IN THE REALM OF  
CLASSICAL GENERAL AND CRYSTAL PHYSICS

BY H. ZOCHER AND C. TÖRÖK

LABORATORIO DA PRODUÇÃO MINERAL, MINISTERIO DA AGRICULTURA, RIO DE JANEIRO,  
BRASIL

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Like thermodynamics and dimensional analysis, symmetry considerations enable one to make predictions without detailed knowledge of the object. At the end of the last century, P. Curie and W. Voigt tried to apply this method, which had proved to be so successful in the morphology of crystals, to physics. Their endeavor was only partially successful. The reason for this fact may be that these great crystal physicists have not adapted and generalized the meaning of symmetry to suit the new field. The mere application of the existing spatial symmetry concept could not be satisfactory, partly because in most physical problems the consideration of asymmetry<sup>1</sup> is more useful than that of symmetry, but principally because most physical problems, in contrast to the morphology of crystals, are concerned not only with spatial structure but also with its relation to time, i.e., movement.

The generalization of the symmetry concept which is necessary for the transition from the morphology of crystals to physics consists in considering transformations involving the time axis in addition to spatial transformations. Symmetry in space is the identity, for the description of a structure, of different coordinate systems which can be obtained by transformations of the space coordinates. Symmetry in relation to time has to be the invariance with respect to transformations of the time coordinate. The transformation may consist in a shifting ( $t' = t + t_0$ ), or in an inversion of time ( $t' = -t$ ), which may be denoted also as reversal of time or as a time reflection. The time inversion is the same operation as letting a movie film run backward. The act of inversion is not a physical act, but the study of the opposite chronological order of the same items. E. Wigner<sup>2</sup> was the first to recognize time transformations as symmetry operations and to apply this kind of operator to problems of quantum mechanics.

In the following we shall be concerned with the inversion of time, not with the shifting of the time axis. We shall speak of time symmetry if the time inversion has no influence upon the sign of the quantity and of time asymmetry if the time inversion causes the sign of the quantity to be reversed. Quantities like linear or rotational velocities are time-asymmetric while a crystal structure is time-symmetric. Quantities related to movement may be time-symmetric, for instance those containing velocity to an even power, such as energy; or those containing acceleration, the second derivative.

In the following we shall consider only rotations and reflections in space and inversion of time but will exclude all space and time translations, as well as relativistic and quantum phenomena. Then we shall introduce the asymmetry concept, which applies both to time-symmetric and time-asymmetric quantities. We shall show that the concept of space-time asymmetry enables one to predict the possibility or impossibility of physical effects, such as pyro-electricity, pyro-magnetism, etc. There is, at present, no certainty about the existence of several such effects which appear to be possible from the point of view of classical crystal physics. In order to judge the possibility or impossibility of physical effects, we assume the axiom that in an equilibrium or in a stationary state no asymmetry can appear which does not exist already in the object or in the influence which is exerted upon it, or in the combination of both.

A physical asymmetry needs always the indication of the relation to time besides the spatial type of asymmetry. The same type of asymmetry exhibiting or not exhibiting invariance with respect to time inversion represent two essentially different asymmetries.

Asymmetry is much more than the counterpart of symmetry. Asymmetry indicates the existence of characteristic differences, whereas symmetry discards characteristic features. Certain physical phenomena are intrinsically related to certain types of asymmetry, whereas certain symmetry elements may exist without necessarily being related to those effects. "C'est la dissymetrie, qui crée le phenomene" stated P. Curie.<sup>3</sup>

The most important application of symmetry considerations in physics studied by P. Curie and W. Voigt is the establishment of the symmetry properties of the electric and magnetic field. Both authors came to the conclusion that the electric field must be considered as a polar vector, the magnetic field as an axial one. Voigt showed that one can conclude from Maxwell's equations that one of these fields must be a polar vector and the other an axial one. The result is obtained immediately by considering the transformation which consists in the multiplication of all space coordinates by the factor  $-1$ . Voigt writes Maxwell's equations in the form given by H. Hertz:

$$\delta A / \delta t = c(\delta Y / \delta z - \delta Z / \delta y); \quad \delta X / \delta t = c(\delta C / \delta y - \delta B / \delta z)$$

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where  $X, Y, Z$  are the electric and  $A, B, C$  the magnetic field components. The decision that the electric field is the polar one is derived from the fact that crystals like tourmaline with polar shape (which is related to polar inner structure) possess an inner electric field. This field is revealed by heating (pyro-electricity) and by compression or dilation (piezo-electricity).

The fact that the magnetic field represents the axial vector can be derived from the Faraday effect which may be called a true circular<sup>4</sup> double refraction: the difference between the refractive indices for light the electric vector of which rotates in the direction of the field-producing current or in the opposite direction.<sup>5</sup>

Disregarding the relation to time, the electric field was assumed to have the same symmetry as linear velocity or polar (hemimorphic) crystals such as tourmaline. Similarly the symmetry of the magnetic field was assumed to be the same as that of angular velocity or circular (paramorphic) crystals, such as apatite, dolomite, etc. On the basis of such a classification one had to expect a series of new physical or crystal physical effects. However, experimental research did not confirm the conclusions arrived at; the expected effects were never found.

It was expected, for instance, that circularly asymmetric crystals could exhibit a constant magnetic moment, in analogy to the electric field in tourmaline. P. Curie<sup>6</sup> made various experiments in order to detect such inherent magnetism but failed to obtain any positive result. W. Voigt<sup>7</sup> gave a rather detailed report of his work to obtain such an effect and concluded that the existence of such an effect—which he called pyromagnetism—is probable. Up to date there does not seem to exist a valid consideration concerning the possibility of the “pyromagnetic” effect.<sup>8</sup>

The whole complex becomes clear, however, if one introduces the concept of asymmetry in relation to time. Applying the operation of time inversion to Maxwell's equations, one sees immediately that one of the two fields is time-symmetric and the other time-asymmetric. In order to decide whether the electric or the magnetic field is time-asymmetric, we consider the same examples as above, the pyro- and piezo-electric crystals and the Faraday effect. The electric field in the polar crystals must be time-symmetric like the crystal structure from which it arises. If we do not assume any further time asymmetry, the magnetic field reverses its sign with time inversion as the rotation of the electric vector of the light in the Faraday effect. This conclusion is to be drawn from the Faraday effect equally well as we derive the enantiomorphic structure of molecules in solution from their optical activity. Summing up, we can say that the electric field is space-asymmetric and time-symmetric, the magnetic field is space-symmetric and time-asymmetric if we call space symmetry and space asymmetry the invariance and the reversal of signs, respectively, under the operation of inversion of space. Having proved the difference in relation to the time we recognize that a magnetic moment cannot arise from a circular crystal structure. *Pyromagnetism and piezomagnetism cannot exist.*

There are other parts of physics in which fruitless experimental investigations could have been avoided by realizing the behavior of the magnetic

field with respect to time inversion. According to the theory of thermal and electrical conductivity of crystals developed by W. Thomson<sup>9</sup> and others, a circular asymmetry of the conductivity seemed to be quite possible. That is to say, the conductivity for clockwise and counterclockwise currents (seen from the same side of the crystal) could be different. Soret<sup>10</sup> and Voigt<sup>11</sup> tried to prove the existence of these "rotational coefficients" in careful experiments. They found no proof for the expected effect. Onsager<sup>12</sup> showed by a new theoretical treatment that such an effect could be expected only under the influence of an external magnetic field or of Coriolis forces.

The circularity of conductivity is a time-asymmetric property. Processes of conduction are not only time-asymmetric, they are irreversible. One direction of time corresponds to the probable course, the other is improbable according to the second law of thermodynamics. Potential differences are time-symmetric. The circular asymmetry consists in the difference of resistance against the clockwise and counterclockwise currents, being probable or not. These currents are naturally time-asymmetric. Hence, a crystal structure, which is time-symmetric, cannot cause a circularly asymmetric conductivity. Such a property can be found only in a system with time-asymmetric circularity, with a mechanical or electrical rotation, with Coriolis forces or magnetic fields. The Hall effect corresponds indeed to circular electric conductivity, the Righi-Leduc effect to a circular thermal conductivity, both produced by a magnetic field.

With the aid of the triasymmetry concept<sup>13</sup> it is possible to extend the above conclusion yet further. The time-asymmetric circularity may be produced by a combination of a time-asymmetric polarity with a time-symmetric enantiomorphism. In other words, circular conductivity may result from a polar conductivity and an enantiomorphic structure. Polar conductivity is a difference of conductivity in two opposite directions. It is evident that polarity of conductivity is a time asymmetric property too, possible only if the object under consideration exhibits a time asymmetric polarity: a linear movement like convection or diffusion of matter, or electric or thermal currents. Contrary to earlier work,<sup>14</sup> polar structure, as it exists in polar crystals such as tourmaline, cannot cause polarity of conductivity. Similarly polarity of electric conductivity and of diffusion cannot be the properties of polar crystals.

The so-called polar conductivity of dry and of electrolytic rectifiers represents no primary property. In the case of primary properties the effect is proportional to the influence ( $i = K \cdot E$ ). If, however, in addition to the linear term, there is a quadratic one ( $i = K_1 \cdot E + K_2 \cdot E^2$ ) the corresponding constant may be called a secondary property. It may appear on account of a change (chemical) in the object proportional to the influence. The effect is therefore proportional to the second power of the influence.

In systems of one crystalline phase only these secondary effects are rather small because an appreciable deformation of the crystal lattice needs very large forces.

In the following we try to make a more positive use of the asymmetry concept, predicting some possible effects. There must exist a new effect, analogous to the Hall effect, in which a circular crystal is substituted for the magnetic field. An electric potential difference applied perpendicularly to the circular axis of such a crystal will produce an electric field perpendicular to the plane through the circular axis and the direction of the applied field. This effect would show that the dielectric constant of merely circular crystals is not represented by a simple second-order tensor, as experiments to date appear to indicate. Molecular considerations appear to support the possibility of this effect.

Two triasymmetric effects can be obtained by means of a circular crystal. In an electric field parallel to the circular axis a circular crystal must acquire enantiomorphism. In this situation the crystal can be expected to show optical activity and mechanical contorsion.

Exerting an enantiomorphic force upon a circular crystal will give this crystal polarity. Hence torsion may produce electric polarization. The electricity produced by torsion could be called "strephelectricity."

The above three effects are possible only in circular crystals. They may deserve some interest because circularity, in contrast to polarity and enantiomorphism, cannot be proved directly. Since circularity is the unique higher asymmetry compatible with a center of symmetry, it is the unique higher asymmetry which manifests itself in Laue diagrams.

The consideration of space-time asymmetry may prove to be useful outside the realm of crystal physics. The process of splitting a magnetic dipole into two free magnetic charges is seen to be impossible if one considers the space-time asymmetry involved in this process. Present-day literature is ambiguous concerning this point. Thus Dirac<sup>15</sup> discusses the reason why the separation of electric charges is so much easier than that of magnetic charges and Ehrenhaft believes to have succeeded in obtaining free magnetic charges.

As already stated, a magnetic momentum corresponds to a time asymmetric circularity and has therefore the character of a simple rotation. As Curie<sup>16</sup> already pointed out, a free magnetic pole would be enantiomorphic. He supposed therefore that optically active substances could possess free magnetism. However, the time asymmetry of the magnetic moment cannot be lost by the process of splitting. Otherwise, the recomposition of the free south and north poles would not furnish the original time-asymmetric moment. Hence, a free magnetic charge would be a time-asymmetric enantiomorphism, a time asymmetric "pseudoscalar" and not a time-symmetric "pseudoscalar" as the enantiomorphic structure of optically active substances is.

A time-asymmetric enantiomorphism occurs in the combination of electric and magnetic fields with parallel lines of force, as proposed by P. Curie<sup>17</sup> in order to produce optical activity in dielectrics. The time-asymmetry of this enantiomorphism, which is indeed a time-asymmetric triasymmetry, does not permit a time-symmetric optical activity. If a magnetic momentum is nothing but what we know about it, i.e., a rotation (of electric charges, of course), then the separation into free magnetic poles is evidently impossible.

It may be useful to point out that, according to the given concept, a magnetic moment, and therefore a spin as well, always corresponds to a real circulating movement and cannot be considered as an intrinsic property without the character of movement. This reemphasizes the fact that static structures in three-dimensional space are not adequate to represent the physical bodies and that the space-time relations are inevitable necessities.

<sup>1</sup> Zocher, H. W. E., and Török, C., *Anais acad. brasil. cienc.*, **20**, 143 (1948).

<sup>2</sup> Wigner, E. P., *Nachr. Ges. Wiss. Goettingen*, **31**, 546 (1932). For more recent literature cf. Luders, G., *Zeits. f. Phys.*, **133**, 325 (1952). cf. also ref 12.

<sup>3</sup> Curie, P., "Oeuvres," Paris, 1908, p. 127.

<sup>4</sup> Circular indicates the asymmetry of axial vectors and paramorphic crystals.

<sup>5</sup> Optical activity is often called a circular double refraction but it would be better labeled as enantiomorphic double refraction, as is easily seen from the comparison of the asymmetry of Faraday effect and optical activity.

<sup>6</sup> Curie, P., *loc. cit.*, p. 133.

<sup>7</sup> Voigt, W., *Lehrbuch der Kristallphysik*, Leipzig, 1928, pp. 261–267.

<sup>8</sup> See, for instance, Bhagavantam, S., and Suryanarayana, D., *Acta Cryst.*, **2**, 24 (1949).

<sup>9</sup> Thomson, W. (Lord Kelvin), *Proc. Roy. Soc. (Edinburgh)*, 123 (1854).

<sup>10</sup> Soret, Ch., *Arch Geneve*, **29**, 4, (1893); **32**, 72, (1894).

<sup>11</sup> Voigt, W., *loc. cit.*, pp. 350, 377.

<sup>12</sup> Onsager, L., *Phys. Rev.*, **37**, 405 (1931).

<sup>13</sup> See Zocher, H., and Török, C., *loc. cit.* It is easily shown that the presence of two higher asymmetries—enantiomorphism, circularity and polarity—along the same direction automatically produces the third. We call the simultaneous presence of the above three asymmetries triasymmetry.

<sup>14</sup> Wooster, W. A., *Crystal Physics*, Cambridge Univ. Press, 1938, p. 7.

<sup>15</sup> Dirac, P. A. M., *Proc. Roy. Soc.*, **A133**, 60 (1931); *Phys. Rev.*, **74**, 817 (1948).

<sup>16</sup> Ehrenhaft, F., *Phys. Rev.* **67**, 207 (1948).

<sup>17</sup> Curie, P., *loc. cit.*, p. 143.

<sup>18</sup> Curie, P., *loc. cit.*, p. 136.

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